## Exercise 52

The *total fertility rate* at time t, denoted by $F(t)$, is an estimate of the average number of children born to each woman (assuming that current birth rates remain constant). The graph of the total fertility rate in the United States shows the fluctuations from 1940 to 2010.

(a) Estimate the values of $F'(1950)$, $F'(1965)$, and $F'(1987)$.

(b) What are the meanings of these derivatives?

(c) Can you suggest reasons for the values of these derivatives?
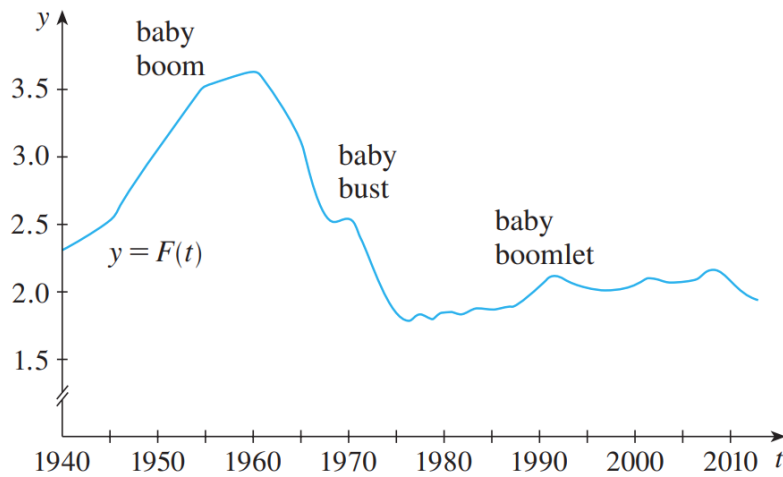
Solution

Draw tangent lines to the curve at $t = 1950$, $t = 1965$, and $t = 1987$. Then determine two points on each of the lines.
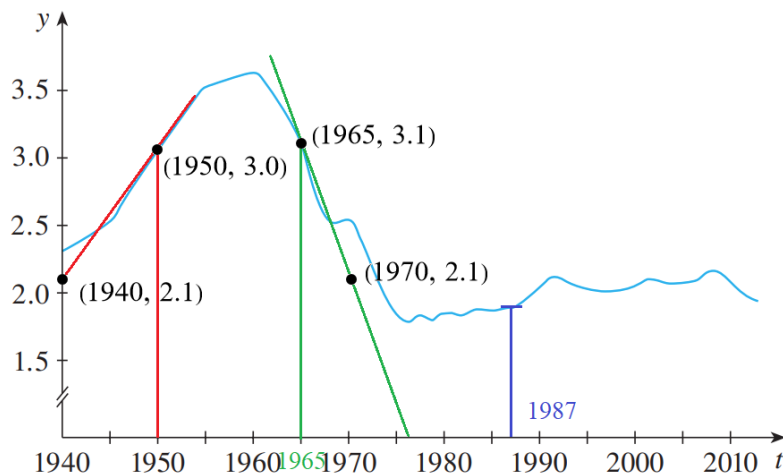
Use the two points on each line to determine its slope, which is the value of the derivative.

$$F'(1950) = m_1 \approx \frac{3.0 - 2.1}{1950 - 1940} = 0.09$$

$$F'(1965) = m_2 \approx \frac{2.1 - 3.1}{1970 - 1965} = -0.2$$

$$F'(1987) = m_3 \approx 0$$

$F'(t)$ represents the rate that the total fertility rate increases as t increases. Its units are children per year. $F'(1950)$ is positive because 1950 is in the baby boom period, $F'(1965)$ is negative because 1965 is in the baby bust period, and $F'(1987)$ is zero because 1987 is between the baby bust and baby boomlet periods.